JMAPS Technical Memorandum 09-06

Stellar Distributions as a Function of I-band Magnitude

Zack Dugan

zachary.dugan@usno.navy.mil

30 April 2010

ABSTRACT

Distribution statistics, including minimum, maximum, average, mode, 2σ maximum, and 2σ minimum, on the number of stars per frame over two sets of 1 million randomly pointed, simulated JMAPS frames are reported as a function of Cousin's I-band magnitude. We find the most populated JMAPS frames lie along the galactic plane, the most heavily populated frame covering the galactic center and including 33,662 stars of I-band magnitude of 17.5 or brighter at the 2σ level. We also find a minimum of 14 stars of I-band magnitude 10.5 or brighter at the 2σ level on which to guide the JMAPS detector.

1. Introduction

Determining general statistics on the number of stars in each JMAPS frame as a function of I-band is critical to setting parameters and requirements for JMAPS, such as the minimum and maximum integration time of the JMAPS detector or the magnitude limits of the mission's guide stars. To develop a general understanding of the distributions of stars of each I-band magnitude over all JMAPS frames, this simulation uses two sets of 1 million frames randomly distributed over the entire sky to obtain representative statistics for JMAPS observations. The USNO NOMAD catalog is used to determine each star's spectral type and then I-band magnitude; each star is then counted in every randomly pointed frame that covers the star. The maximum, minimum, average, mode, 2σ maximum, and 2σ minimum number of stars per frame are recorded as a function of both magnitude and cumulative magnitude.[1] This process is repeated for the second set of 1 million randomly pointed frames to verify the results from the first simulation.

[1]Cumulative magnitude refers to a magnitude bin plus all brighter magnitude bins.

To validate the locations of the most populated JMAPS frames, those frames are plotted along with the galactic plane.

2. Methodology

The first task for each star is to determine its I-band magnitude using information provided in NOMAD. First, the spectral type is determined to calculate each star's V-I value and then the I-band magnitude. Many of the entries in NOMAD include bad data on existing stars, and in some cases, data on artifacts of other stars. The purpose of the initial steps of this part of the algorithm is to ignore artifacts and stars with bad data. Data from the 2MASS catalog (J, H, and K-band photometry) are used because information from 2MASS is generally more accurate than other parts of NOMAD and because 2MASS contains fewer artifacts and stars with bad data. Initially, if a NOMAD entry does not have a corresponding 2MASS identification number, it is rejected. To determine each star's spectral type, the V-K value is first used, assuming all stars to be Zero Age Main Sequence Stars (ZAMS). This value is not used if no K-band data is available or if the V-K value falls outside the limits provided in Allen's Astrophysical Quantities (1) and Modern Astrophysics (2). If the V-K value is valid, the star's spectral type is classified. If not, the V-J value is then used, then V-H, and finally B-V. All stars of V-band magnitude of 10.5 or brighter are also required to have a Tycho-2 catalog number to ensure the validity of the entry. The V-band magnitudes of all the ignored stars are recorded in Table 1.

Next, the spectral type is used to calculate the V-I value and then the I-band magnitude of the star in question. The following derivation of the relationship between the apparent I-band magnitude and the V-band magnitude begins with the definitions of the zero point flux densities for each wave band. The zero-point flux ($f_\lambda{}'$) density for the Johnson V-band (500-590 nm) is defined as the photon flux density for a zero-magnitude, A0-type star (Dorland 2007):

$$f_\lambda{}'_v = 9.969 \times 10^3 ph \times s^{-1} \times cm^{-2} \times nm^{-1} \tag{1}$$

where the aperture area is in units of cm^{-2} and the bandwidth is in units of nm^{-1}. The zero-point flux ($f_\lambda{}'_i$) density for the Cousins I-band (697.5-896nm) is calibrated so that the V-I index is zero for an A0 star as it theoretically should be.

Apparent magnitude is defined as the following, and equation 2, rearranged, becomes equation 3:

$$m_v = -2.5 \times log\left(\frac{f_v}{{f_\lambda}'_v}\right) \tag{2}$$

$$f_v = {f_\lambda}'_v \times 10^{(-0.4 \times m_v)} \tag{3}$$

Likewise:

$$m_i = -2.5 \times log\left(\frac{f_i}{{f_\lambda}'_i}\right) \tag{4}$$

We define the fraction for each spectral type as the ratio of the flux in the Cousins' I-band over the flux in the V-band:

$$fraction_{SpectralType} = \frac{f_i}{f_v} \tag{5}$$

We can substitute in:

$$m_i = -2.5 \times log\left(\frac{f_v \times fraction_{SpectralType}}{{f_\lambda}'_i}\right) \tag{6}$$

and substitute in equation 4 then rearrange to obtain:

$$m_i = m_v - 2.5 * log\left(\frac{{f_\lambda}'_{(v)}}{{f_\lambda}'_{(i)}}\right) - 2.5 * log\left(fraction_{SpectralType}\right) \tag{7}$$

The last two elements of equation 7 combine to become a specific constant for each spectral type, as shown in equation 8:

$$m_i = m_v - constant_{SpectralType} \tag{8}$$

This specific constant is equal to -(B-I)$_{SpectralType}$. Thus, to determine the apparent I-band magnitude, the fraction between the I-band flux and the V-band flux is calculated for 27 spectral types from O5-M6 using spectra from Pickles 1998. Figure 1 shows plots of the flux in the V and I-bands for an A7 star, and Figure 2 shows the same fluxes for spectral types F0, G0, K0, and M0. Table 1 shows the values of the I-band to V-band flux ratio as a function of spectral type as well as the constants from equation 8 as a function of spectral type.

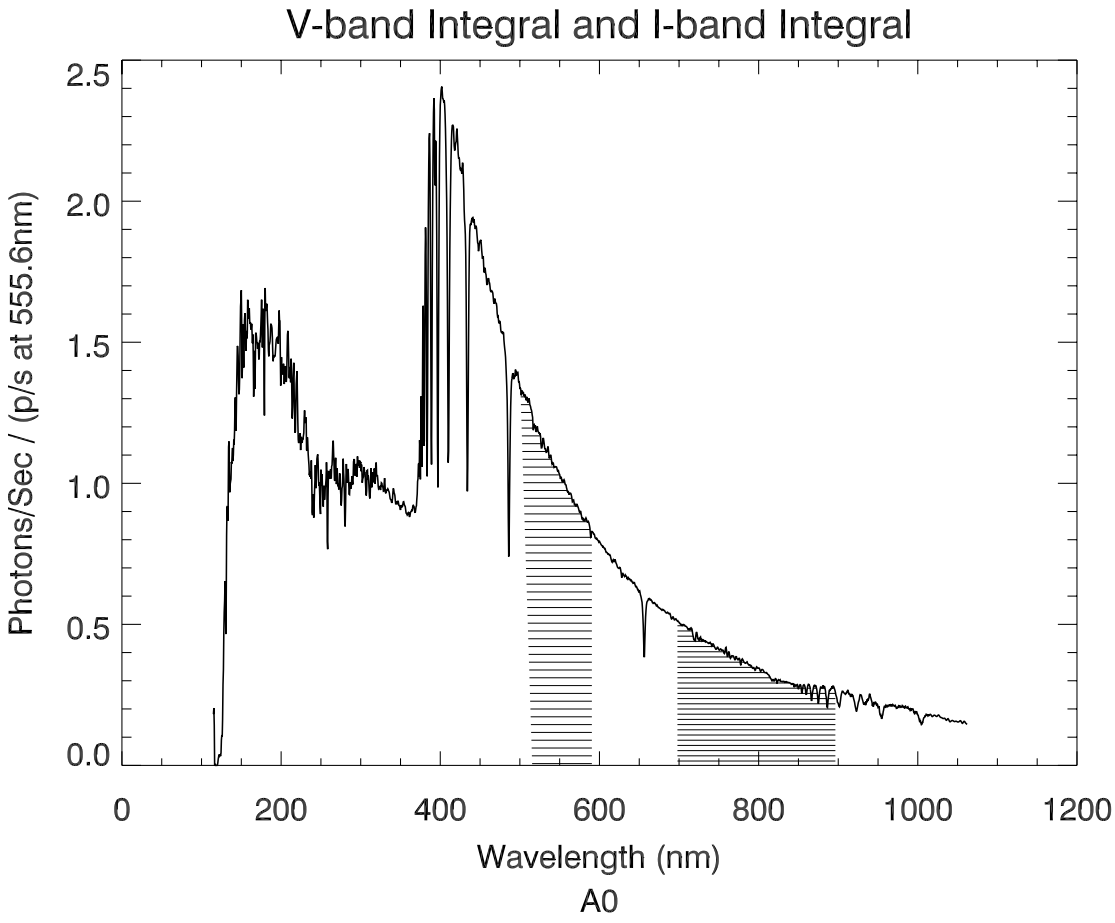

Figure 1: Flux Ratio Between V and I-band for an A0 Star

With the star's I-band magnitude calculated, the next step is to determine which of the 1 million randomly pointed frames include the star in question. First, the stars right ascension and declination are converted into x, y, and z components of a unit vector. Each of the random pointings are given as unit vectors. To reduce the total run time of the program, circular frames are assumed with areas equivalent to the square frames 1.24 degrees on a side that JMAPS will actually observe. With enough iterations, the effects caused by the difference in frame shape diminish until they are negligible. The dot product of the random pointing unit vector and the star's unit vector is calculated, and if the result was less than the radius of the circular frame, assuming flat geometry, then the star is within the limits of the frame. The process was repeated with a second set of 1 million randomly pointed frames

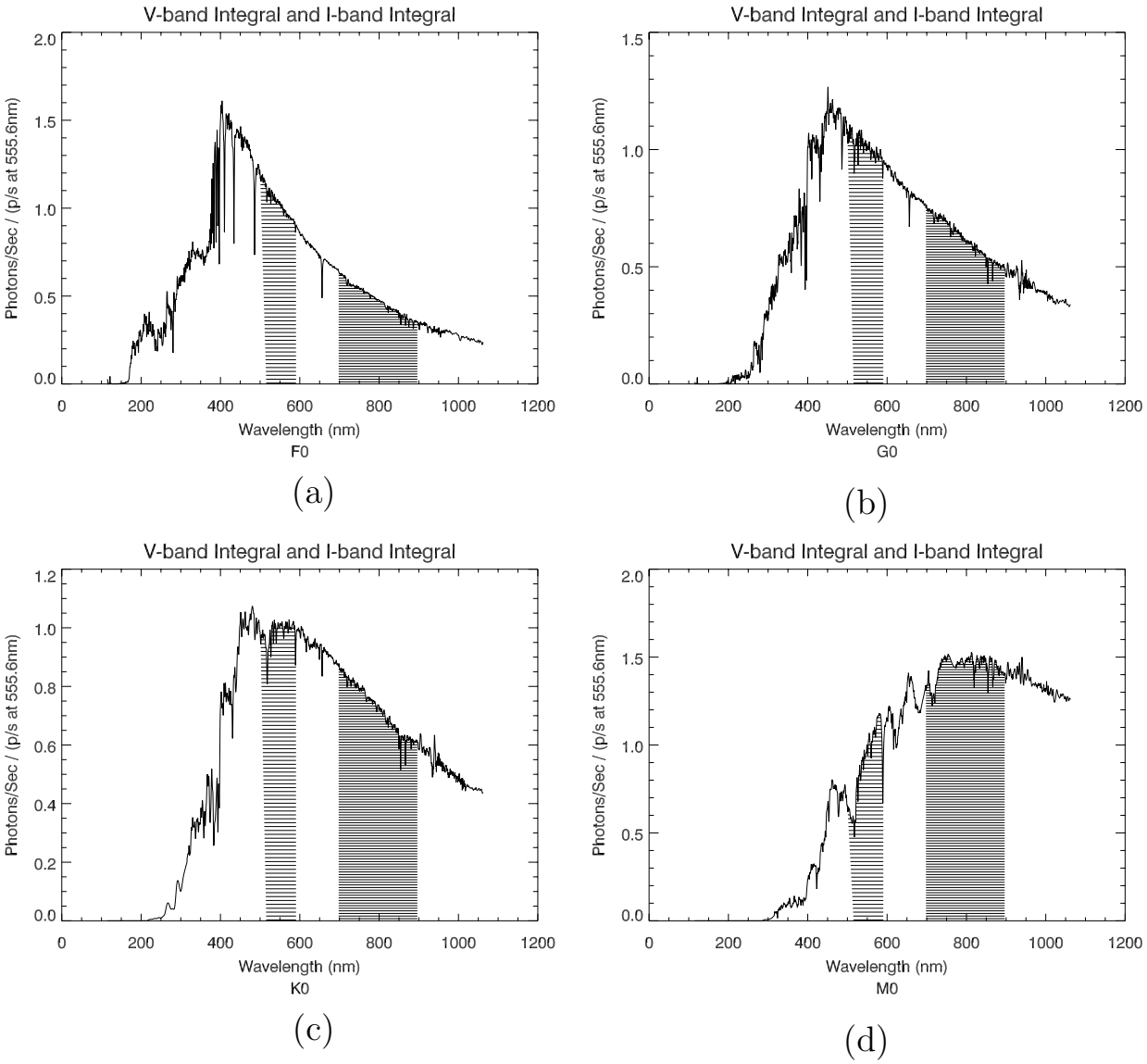

Figure 2: Flux Ratios Between the V-band and I-band for an F0, G0, K0, M0 (a-d, respectively).

Table 1: I-Band : V-Band Flux Ratios and Corresponding Equation Constants

Spectral Type	I-Band:V-Band Flux Ratio	Corresponding Equation Constant
O5	0.345	0.361
09	0.346	0.358
B0	0.359	0.318
B3	0.413	0.165
B5	0.442	0.094
B8	0.457	0.058
A0	0.481	-0.000
A2	0.499	-0.040
A5	0.550	-0.144
A7	0.595	-0.230
F0	0.669	-0.358
F2	0.727	-0.448
F5	0.754	-0.487
F8	0.843	-0.608
G0	0.885	-0.661
G2	0.927	-0.711
G5	0.945	-0.732
G8	1.016	-0.811
K0	1.070	-0.867
K3	1.350	-1.120
K4	1.519	-1.248
K5	1.711	-1.377
K7	2.097	-1.598
M0	2.360	-1.726
M1	2.722	-1.881
M2	3.144	-2.038
M3	4.541	-2.437
M4	6.186	-2.772
M5	8.778	-3.152
M6	14.854	-3.723

Note: The corresponding equation constant is equal to -(B-I).

to verify the distribution results from the first simulation.

To further reduce the run time of the program, each star is only tested on frames in a reasonably nearby neighborhood. A heap sort was implemented on the array containing the 1 million random pointings to sort each pointing by its x-component, so that each star would only be checked on pointing unit vectors with similar x-components.

3. Data

In both sets of randomly pointed JMAPS frames, exactly 157,122,106 stars were successfully tabulated, resulting in a 99.6% scan rate. The percentages of the total entries rejected by not having a 2MASS entry, having bad spectral classification data (V-K, V-J, V-H, and B-V), or by having no Tycho-2 entry for a star with a V-band magnitude of 10.5 or brighter were 0.268%, 0.135%, and 0.002%, respectively.

Table 2 shows the total number of stars per I-band magnitude, binned from -0.5 to 0.5 etc., and the total cumulative number of stars of an I-band magnitude and all brighter magnitudes, meaning the number of stars brighter than I-band magnitude 0.5, etc.

Table 2: Stars and Cumulative Stars per I-Band Magnitude

I-Band Magnitude	Stars	Stars Per Average JMAPS Frame	Cumulative Stars	Cumulative Stars per Average JMAPS Frame
0	18	0.00	18	0.00
1	25	0.00	43	0.00
2	82	0.00	125	0.00
3	492	0.02	617	0.02
4	1912	0.07	2529	0.09
5	6345	0.24	8874	0.33
6	19487	0.73	28361	1.06
7	55747	2.08	84108	3.14
8	150791	5.62	234899	8.76
9	390009	14.54	624908	23.29
10	968563	36.10	1593471	59.40
11	2442797	91.06	4036268	150.45
12	6364794	237.25	10401062	387.70
13	14984533	558.55	25385595	946.25
14	30111971	1122.43	55497566	2068.68
15	52714686	1964.95	108212252	4033.63
16	47794012	1781.53	156006264	5815.16
17	1115809	41.60	157122073	5856.75

Table 3 shows the number and percentage of ignored objects as a function of V-band magnitude.

Table 4 shows general statistics on the number of stars per I-band magnitude, again binned from -0.5 to 0.5, etc. The 2σ minimum refers to the number of stars per JMAPS

Table 3: Ignored Objects as a Function of V-Band Magnitude

V-Band	Number Objects	Percent of Total Objects
0	0	0.000
1	2	0.000
2	28	0.004
3	12	0.002
4	83	0.013
5	584	0.091
6	2443	0.382
7	9403	1.472
8	20724	3.244
9	18206	2.850
10	17636	2.761
11	22361	3.500
12	40757	6.380
13	77362	12.110
14	108957	17.056
15	92994	14.557
16	128515	20.118
17	98753	15.459

frame that is less than the number of stars in 97.72% of the total frames, and likewise, the 2σ maximum refers to the number of stars per JMAPS frame that is greater than the number of stars in 97.72% of the total frames. For example, in Table 4, at the 9^{th} magnitude, 97.72% of the frames have 2 or more stars per frame and 97.72% of the frames have 46 or fewer stars per frame. Table 5 shows the exact same data for the cumulative number of stars per JMAPS frame, meaning the number of stars of a certain I-band magnitude or brighter. Tables 6 and 7 show general statistics on the non-cumulative and cumulative number of stars per JMAPS frame from the second set of 1 million randomly pointed frames. Tables 8 and 9 show the locations of the most populated JMAPS frames as a function of I-band magnitude for the first and second set of randomly pointed frames.

4. Analysis

Figure 3 shows the most cumulatively populated JMAPS frames of magnitudes 0-17 along with the galactic plane and the galactic center. The majority of the most populated frames lie along the galactic plane, with many of the dimmer magnitudes falling on the galactic center itself. The magnitudes whose most populated frames are not along the galactic plane have very few stars per frame. A number of these frames have that same maximum number of stars per frame, although only one frame is plotted for each magnitude

Table 4: Stars per Simulated JMAPS Frame as a function of I-Band Magnitude

I-Band Magnitude	Min. Stars per Frame	2 σ Min. Stars per Frame	Mode Stars per Frame	Average Stars per Frame	2 σ Max Stars per Frame	Max. Stars per Frame
0	0	0	0	0.002	0	1
1	0	0	0	0.001	0	1
2	0	0	0	0.003	0	1
3	0	0	0	0.018	0	2
4	0	0	0	0.071	1	5
5	0	0	0	0.237	2	7
6	0	0	0	0.725	3	11
7	0	0	1	2.080	7	21
8	0	0	3	5.622	16	44
9	0	2	6	14.539	46	118
10	0	6	12	36.117	142	516
11	3	14	22	91.044	447	3186
12	0	32	46	237.193	1316	12560
13	0	74	95	558.438	3276	28159
14	0	128	155	1122.498	6951	49242
15	0	195	262	1966.351	11942	48825
16	0	130	196	1781.341	10746	22757
17	0	0	2	41.628	312	1498

Note: This data shows statistics based on the first set of 1 million randomly pointed JMAPS frames.

Table 5: Cumulative Stars per Simulated JMAPS Frame

I-Band Magnitude	Min. Stars per Frame	2 σ Min. Stars per Frame	Mode Stars per Frame	Average Stars per Frame	2 σ Max Stars per Frame	Max. Stars per Frame
0	0	0	0	0.002	0	1
1	0	0	0	0.003	0	1
2	0	0	0	0.006	0	2
3	0	0	0	0.024	1	2
4	0	0	0	0.095	1	6
5	0	0	0	0.332	2	8
6	0	0	0	1.057	4	16
7	0	0	2	3.137	9	35
8	0	1	5	8.759	23	71
9	0	5	11	23.298	68	181
10	3	14	21	59.415	208	646
11	12	31	44	150.459	652	3822
12	22	67	91	387.652	1965	16365
13	23	145	176	946.091	5255	43864
14	23	278	315	2068.589	12205	91541
15	28	479	614	4034.939	24178	122652
16	31	625	827	5816.280	33435	123482
17	31	628	830	5857.909	33626	123496

Note: This data shows statistics based on the first set of 1 million randomly pointed JMAPS frames.

Table 6: Stars per Simulated JMAPS Frame as a function of I-Band Magnitude

I-Band Magnitude	Min. Stars per Frame	2 σ Min. Stars per Frame	Mode Stars per Frame	Average Stars per Frame	2 σ Max Stars per Frame	Max. Stars per Frame
0	0	0	0	0.002	0	1
1	0	0	0	0.001	0	1
2	0	0	0	0.003	0	2
3	0	0	0	0.018	0	2
4	0	0	0	0.071	1	6
5	0	0	0	0.238	2	7
6	0	0	0	0.726	3	10
7	0	0	1	2.078	7	19
8	0	0	3	5.621	16	46
9	0	2	6	14.537	46	123
10	0	6	12	36.108	142	500
11	3	14	22	91.029	447	3189
12	0	32	46	237.120	1312	12552
13	0	74	97	558.418	3271	28188
14	0	128	154	1122.288	6932	49401
15	0	195	256	1964.968	11902	48778
16	0	131	196	1781.967	10761	22787
17	0	0	2	41.598	310	1479

Note: This data shows statistics based on the second set of 1 million randomly pointed JMAPS frames.

Table 7: Cumulative Stars per Simulated JMAPS Frame

I-Band Magnitude	Min. Stars per Frame	2 σ Min. Stars per Frame	Mode Stars per Frame	Average Stars per Frame	2 σ Max Stars per Frame	Max. Stars per Frame
0	0	0	0	0.002	0	1
1	0	0	0	0.003	0	1
2	0	0	0	0.006	0	2
3	0	0	0	0.024	1	2
4	0	0	0	0.095	1	7
5	0	0	0	0.332	2	8
6	0	0	0	1.059	4	16
7	0	0	2	3.136	9	32
8	0	1	5	8.757	23	67
9	0	5	11	23.294	68	179
10	4	14	22	59.402	208	649
11	11	31	41	150.431	653	3815
12	24	67	89	387.550	1961	16357
13	25	145	168	945.968	5241	43842
14	25	278	325	2068.257	12182	91964
15	31	479	614	4033.225	24105	123010
16	32	626	803	5815.192	33474	123916
17	32	629	707	5856.789	33662	123931

Note: This data shows statistics based on the second set of 1 million randomly pointed JMAPS frames.

Table 8: Locations of Most Populated JMAPS Frames

I-Band	Right Ascension (hours)	Declination (degrees)
0	12.147	-22.068
1	12.881	3.306
2	7.008	-28.525
3	11.571	-31.263
4	3.799	24.302
5	11.642	-61.224
6	11.642	-61.224
7	11.594	-60.996
8	10.759	-59.710
9	11.116	-58.957
10	18.032	-28.597
11	18.026	-28.673
12	18.032	-28.597
13	18.032	-28.597
14	18.077	-27.884
15	18.200	-27.578
16	18.200	-27.578
17	18.200	-27.578

Note: This data shows statistics based on the first set of 1 million randomly pointed JMAPS frames.

Table 9: Locations of Most Populated JMAPS Frames

I-Band	Right Ascension (hours)	Declination (degrees)
0	12.170	-21.958
1	12.883	3.410
2	7.012	-28.478
3	11.566	-31.242
4	3.794	24.321
5	11.634	-61.259
6	11.613	-61.219
7	11.613	-61.219
8	10.694	-59.633
9	11.109	-59.047
10	18.011	-28.529
11	18.031	-28.561
12	18.031	-28.561
13	18.031	-28.561
14	18.066	-27.980
15	18.204	-27.599
16	18.204	-27.599
17	18.204	-27.599

Note: This data shows statistics based on the second set of 1 million randomly pointed JMAPS frames.

in Figure 3. The rest of the most populated frames of I-band magnitudes 0-3 are shown in Figure 4 a-d. Figures 5 and 6 show the same data that Figures 3 and 4 do, except on the second set of 1 million randomly pointed JMAPS frames.

To further demonstrate the density of stars along the galactic plane, all stars of magnitudes 5.5 and dimmer included in this study were plotted in Figure 7.

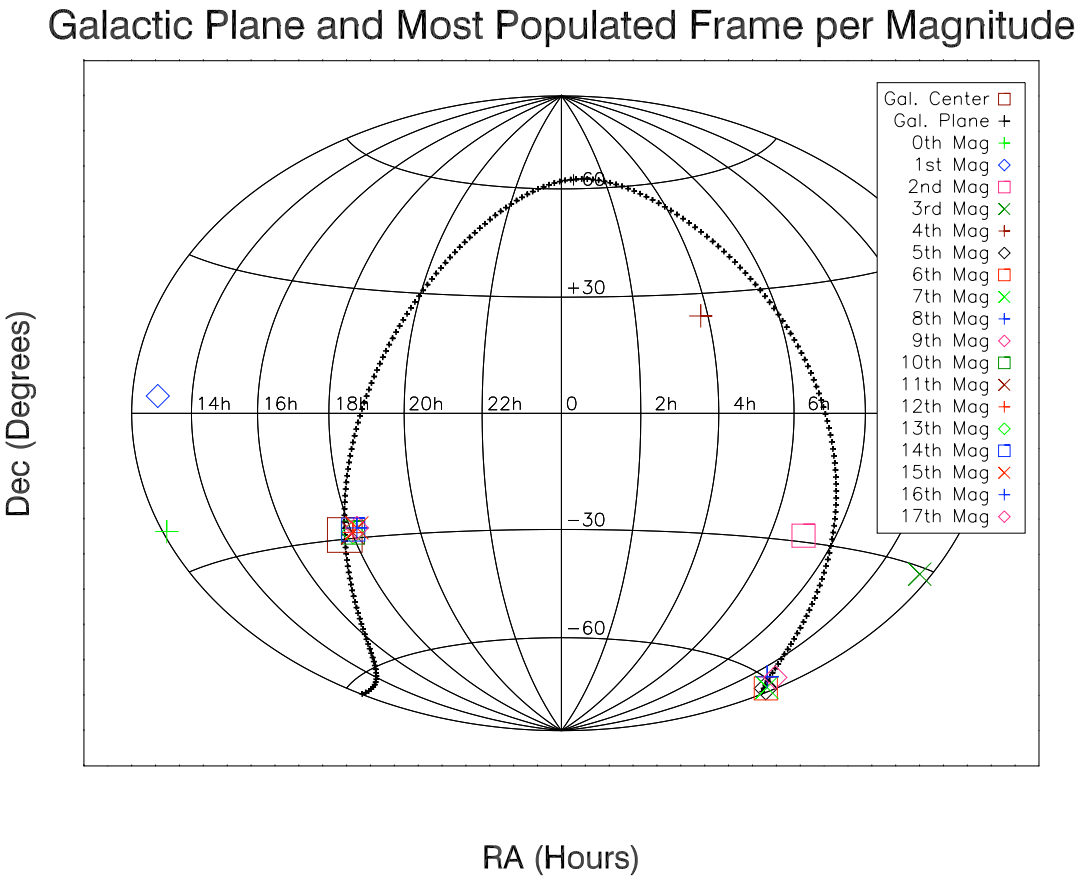

Figure 3: Most Populated JMAPS Frames
for 1^{st} Set of Randomly Pointed JMAPS Frames

As expected, the majority of the most populated frames are on or close to the galactic plane, as shown in Figure 3. The most populated frames for magnitudes 10-17 cover the galactic center, while the most populated frames for magnitudes 5-9 lie on the same part of

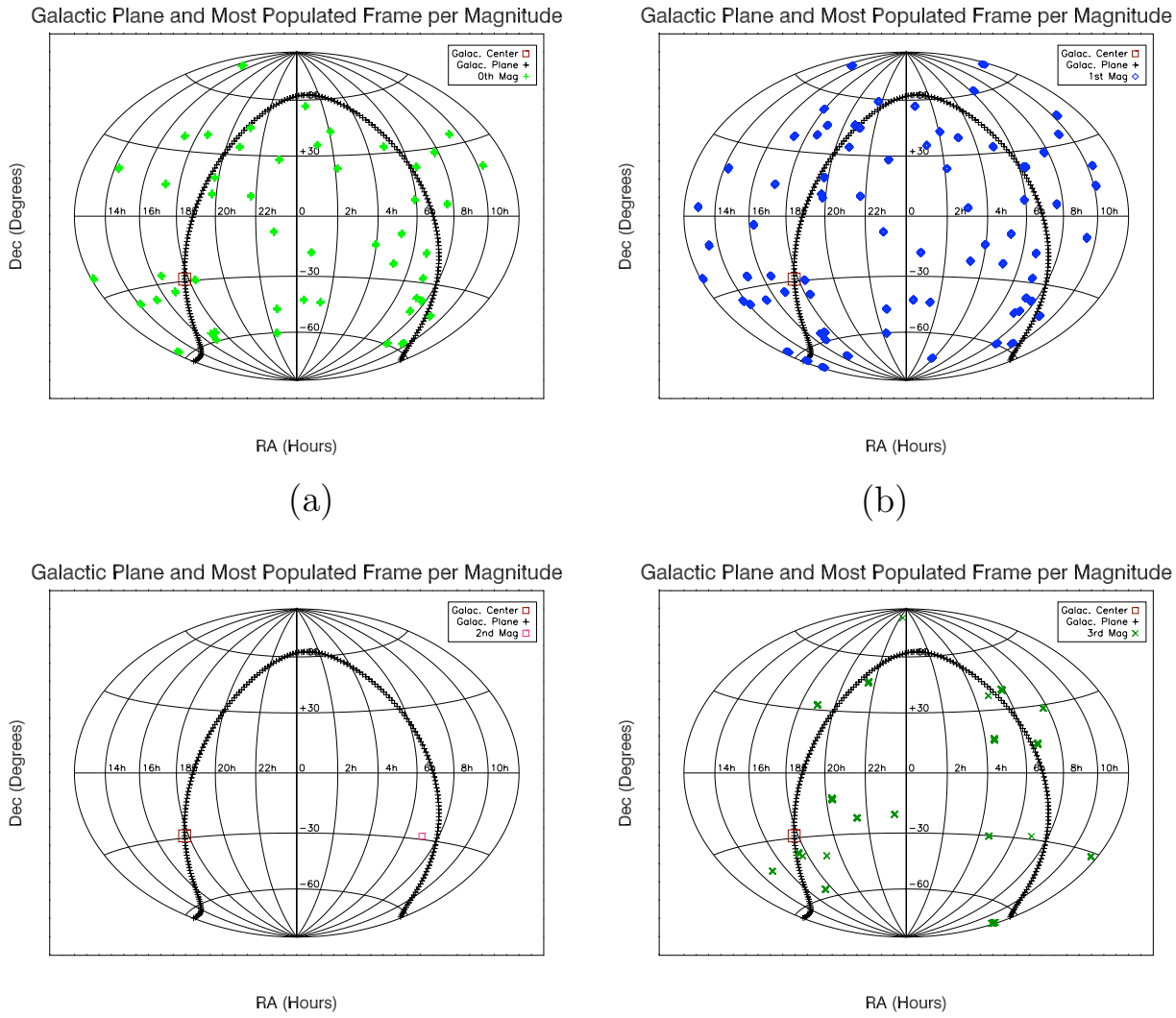

Figure 4: Most Cumulatively Populated JMAPS Frames of Magnitudes 0-3 (a-d, respectively) for 1^{st} Set of Randomly Pointed JMAPS Frames.

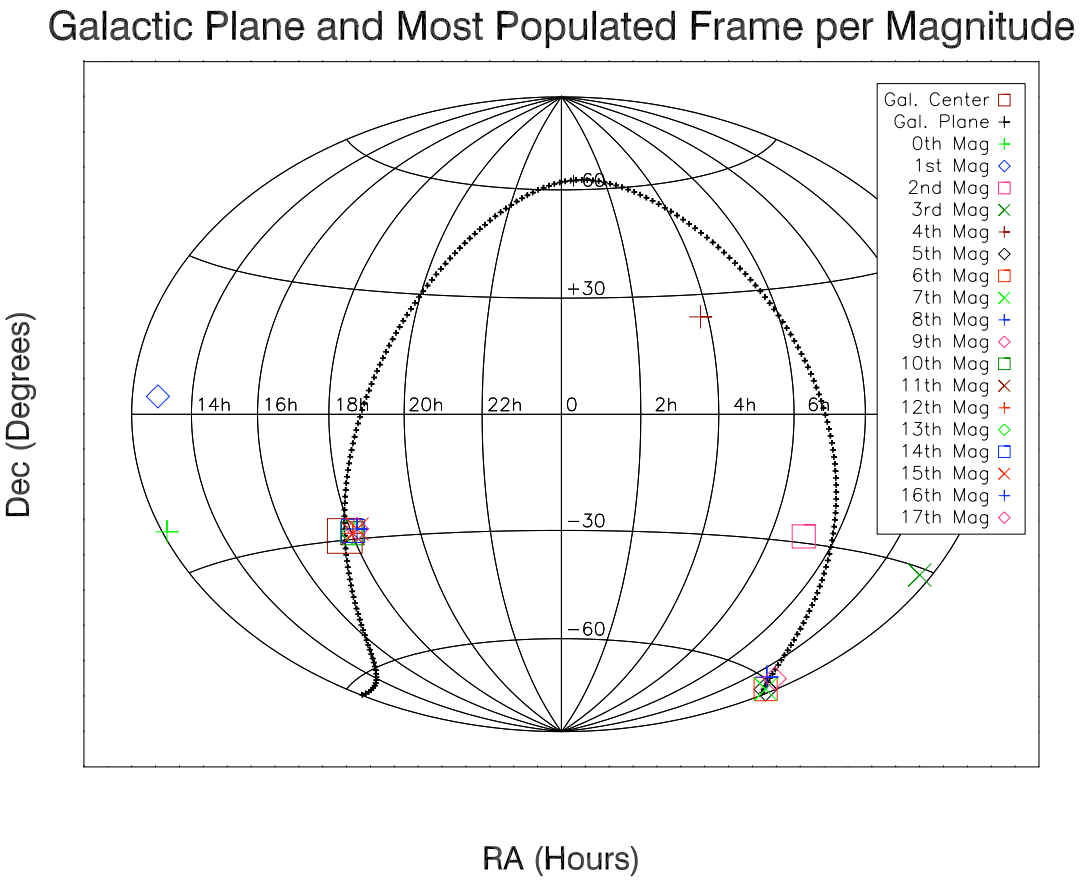

Figure 5: Most Populated JMAPS Frames
for 2^{nd} Set of Randomly Pointed JMAPS Frames

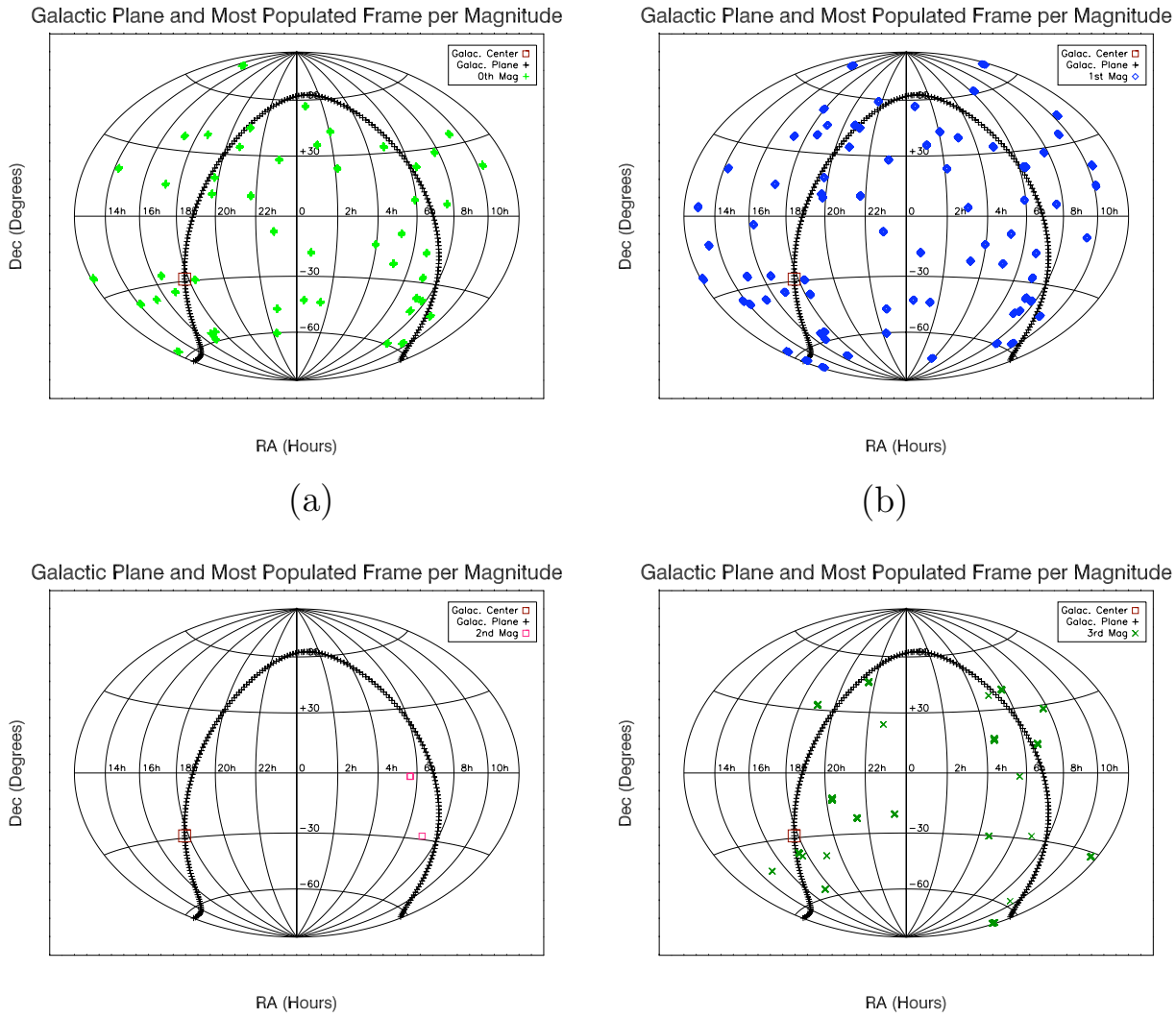

Figure 6: Most Cumulatively Populated JMAPS Frames of Magnitudes 0-3 (a-d, respectively) for 2^{nd} Set of Randomly Pointed JMAPS Frames.

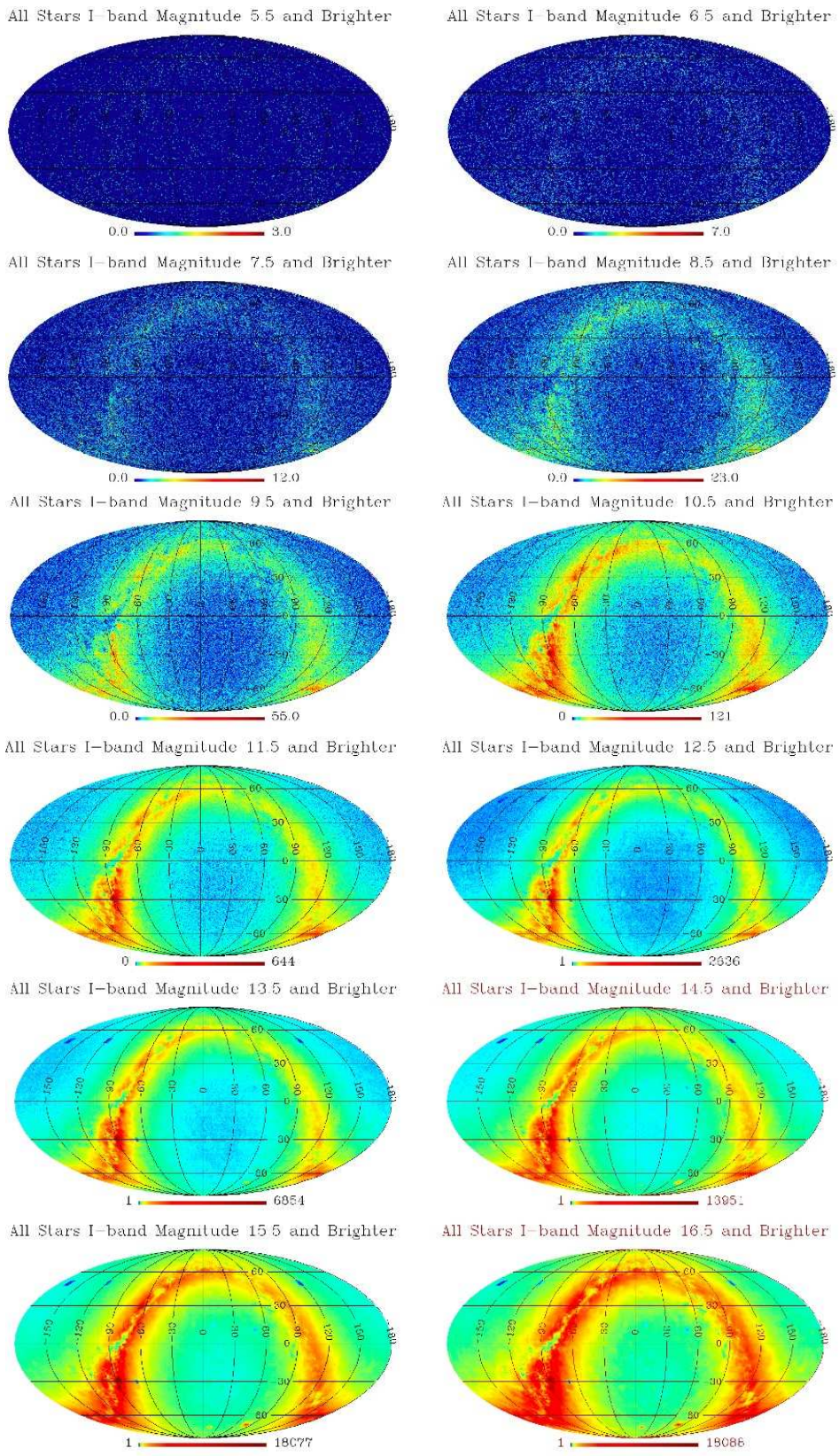

Figure 7: Locations of Cumulatively Bright Stars of Magnitudes 5-15

the galactic plane in the southern hemisphere. Likewise, the majority of the rejected NOMAD entries have dimmer magnitudes, illustrating the limits of accuracy of the NOMAD catalog at the dimmer magnitudes. Because most stars' I-band magnitude is brighter (up to 3 magnitudes brighter) than its V-band magnitude, stars with I-band magnitudes of about 10 are drawing on V-band data from magnitudes 10-14, which is where the comprehensiveness and accuracy of the NOMAD catalog begins to drop off heavily. Both the minimum and maximum number of stars per JMAPS frame as a function of a single I-band magnitude (Table 4) reflect this same drop off. The minimum is 0 for magnitudes 0-12 and 14-17, demonstrating the lack of completeness of NOMAD for I-band magnitudes 13 and dimmer. Similarly, the mode, average, 2σ maximum, and the maximum number of stars per frame decrease significantly after 16^{th} magnitude in the non-cumulative tabulations.

5. Conclusions

The excellent agreement between the statistics and positions of the most populated frames derived from the two separate sets of 1 million randomly pointed JMAPS frames shows that the actual locations of the randomly pointed frames did not have a significant effect on the results, as shown in Tables 4, 5, 6, and 7 and Figures 3, 4, 5, and 6.

All the data from Table 5 shows a large discrepancy between actual maximums and actual minimums and their respective 2σ values. Because the average, mode, and 2σ maximum values are all significantly smaller than the maximum, more importance should be placed on the 2σ maximum than the actual maximum. Similarly, the most populated JMAPS frames of I-band magnitudes 11-17 all cover the galactic center, which has a much denser population of stars as demonstrated by the 2σ maximums. The most populated frame including all stars of I-band magnitude 17.5 or brighter has a maximum of 123,496 stars per frame and a 2σ maximum of 33,626 stars per frame.

The algorithm used for this study does leave room for systematic extinction problems. The V-band is more susceptible to extinction and reddening than the K, J, or H-bands, and the V-K, V-J, and V-H values will not be entirely accurate for spectral classification as a result. Miscalculation in this process would in turn propagate errors to the calculated I-band magnitudes. The additional use of the B-V value includes some stars suffering from extinction that would otherwise be ignored because the B and V-bands suffer from similar amounts of extinction. However, use of the B-V value does not eliminate the systematic errors in the I-band calculations.

Further analysis should be done to account for reddening and extinction in the determination of the I-band magnitudes.

6. References

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