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Stellar Distributions as a Function of I-band Magnitude from SuperCOSMOS and a New B-Band Algorithm

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ABSTRACT

Distribution statistics, including minimum, maximum, average, mode, 2σ maximum, and 2σ minimum, on the number of stars per frame over two sets of 1 million randomly pointed, simulated JMAPS frames are reported as a function of Cousin's I-band magnitude to complement Dugan TM 09-06. We examine the same 2MASS and NOMAD data as in TM 09-06, but use a new algorithm based largely on B-band photometric data in place of V-band photometric data to verify the results and examine the effects of reddening in TM 09-06. We also examine new I-band data from SuperCOSMOS to corroborate results from the first two algorithms. We find good overall agreement between all three, particularly for critical JMAPS parameters, such as guide stars.

1. Introduction

Determining general statistics on the number of stars in each JMAPS frame as a function of I-band is critical to setting parameters and requirements for JMAPS, such as the minimum and maximum integration time of the JMAPS detector or the magnitude limits of the mission's guide stars. To develop a general understanding of how reddening may effect distribution statistics as a function of magnitude I-band magnitude in JMAPS TM 09-06, which was based on V-band color indices, this simulation utilizes B-band color indices instead. Because B-band color indices should be more susceptible to reddening, if the two methods yield the same results, reddening is not an issue, and vice versa.

To obtain representative statistics for JMAPS observations, this simulation uses the first of two sets of 1 million frames randomly distributed over the entire sky from TM 09-06. The

USNO Naval Observatory Merged Astrometric Dataset (Zacharias et al. 2004; NOMAD) catalog is used to determine each star's spectral type and then I-band magnitude, as in TM 09-06. Each star is then counted in every randomly pointed frame that covers the star. The maximum, minimum, average, mode, 2σ maximum, and 2σ minimum number of stars per frame are recorded as a function of both magnitude and cumulative magnitude.[1] To verify important JMAPS parameters, a second algorithm, similar to the first used in TM 09-06, is used to analyze data from the Two Micron All Sky Survey (Skrutskie et al. 2003, 2MASS) and NOMAD, and the results are compared with observed I-band data from the SuperCOSMOS Sky Survey (Hambley et al. 2001).

All observed I-band data was taken from the SuperCOSMOS Science Archive, which holds the object catalogue data extracted from scans of photographic Schmidt survey plates. At 4 terabytes, the database contains more than 6.3 billion individual object detections, merged into 1.9 billion multi-colour sources. SuperCOSMOS complements 2MASS and NOMAD because it contains all-sky I-band data, which 2MASS and NOMAD lack. JMAPS, however, will observe in Cousins I-band, thus the SuperCOSMOS data are crucial and can be used to compare direct observational results with simulations based on 2MASS and NOMAD.

2. Methodology

The previous algorithm from Dugan TM 09-06, which will henceforth be referred to as the V-band algorithm and is documented in TM 09-06, was potentially susceptible to systematic interstellar reddening induced errors. In order to test the effects of reddening on the results from TM 09-06, a newer algorithm based on B-band data instead of V-band data was implemented.

The initial task for the newer B-band algorithm is to determine each star's I-band magnitude using information provided in NOMAD. First, the spectral type is determined to calculate each star's B-I value and then the I-band magnitude. Many of the entries in NOMAD include bad data on existing stars, and in some cases, data on artifacts of other stars. To isolate and avoid these artifacts and other inaccurate information, data from the 2MASS catalog (J, H, and K-band photometry) are used in combination with NOMAD because 2MASS is generally more accurate than other parts of NOMAD and because 2MASS contains fewer artifacts and stars with bad data. Initially, if a NOMAD entry does not have a corresponding 2MASS identification number, it is rejected. To determine each star's spectral

[1] Cumulative magnitude refers to a magnitude bin plus all brighter magnitude bins.

type, the B-K value is first used, assuming all stars to be Zero Age Main Sequence Stars (ZAMS). This value is not used if no K-band data is available or if the B-K value falls outside the limits provided in Allen's Astrophysical Quantities (1) and Modern Astrophysics (2). If the B-K value is valid, the star's spectral type is classified. If not, the B-J value is then used, then B-H, and finally B-V. All stars of V-band magnitude of 10.5 or brighter are also required to have a Tycho-2 catalog number to ensure the validity of the entry.

Next, the spectral type is used to calculate the B-I value and then the I-band magnitude of the star in question. The following derivation of the relationship between the apparent I-band magnitude and the B-band magnitude begins with the definitions of the zero point flux densities for each wave band. The zero-point flux density ($f_{\lambda}{'}_b$) for the Johnson B-band (400-490 nm) is defined as the B-band photon flux density for a zero-magnitude, A0-type star. The zero-point flux density ($f_{\lambda}{'}_i$) for the Cousins I-band (697.5-896nm) is calibrated so that the V-I index is zero for an A0 star as it theoretically should be.

Apparent magnitude is defined as the following, and equation 1, rearranged, becomes equation 2:

$$m_b = -2.5 \times log\left(\frac{f_b}{f_{\lambda}{'}_b}\right) \tag{1}$$

$$f_b = f_{\lambda}{'}_b \times 10^{(-0.4 \times m_b)} \tag{2}$$

Likewise:

$$m_i = -2.5 \times log\left(\frac{f_i}{f_{\lambda}{'}_i}\right) \tag{3}$$

We define the fraction for each spectral type as the ratio of the flux in the Cousins' I-band over the flux in the B-band:

$$fraction_{SpectralType} = \frac{f_i}{f_b} \tag{4}$$

We can substitute in:

$$m_i = -2.5 \times log\left(\frac{f_b \times fraction_{SpectralType}}{f_{\lambda}{'}_i}\right) \tag{5}$$

and substitute in equation 3 then rearrange to obtain:

$$m_i = m_b - 2.5 * log\left(\frac{f_{\lambda}{'}_{(b)}}{f_{\lambda}{'}_{(i)}}\right) - 2.5 * log\left(fraction_{SpectralType}\right) \tag{6}$$

The last two elements of equation 6 combine to become a specific constant for each spectral type, as shown in equation 7:

$$m_i = m_b - constant_{SpectralType} \tag{7}$$

Thus, to determine the apparent I-band magnitude, the fraction between the I-band flux and the B-band flux is calculated for 27 spectral types from O5-M6 using spectra from Pickles 1998. Table 1 shows the values of the I-band to B-band flux ratio as a function of spectral type as well as the constants from equation 7, which equal -(V-I), as a function of spectral type. Table 2 shows the same values for the V-band algorithm documented in TM 09-06.

Table 1: I-Band : B-Band Flux Ratios and Corresponding Equation Constants

Spectral Type	I-Band:B-Band Flux Ratio	Corresponding Equation Constant
O5	0.127	0.684
09	0.133	0.637
B0	0.137	0.608
B3	0.175	0.338
B5	0.195	0.221
B8	0.206	0.162
A0	0.239	-0.000
A2	0.251	-0.054
A5	0.310	-0.281
A7	0.346	-0.401
F0	0.425	-0.625
F2	0.501	-0.802
F5	0.546	-0.897
F8	0.652	-1.089
G0	0.697	-1.162
G2	0.770	-1.269
G5	0.820	-1.337
G8	0.923	-1.466
K0	0.989	-1.541
K3	1.388	-1.909
K4	1.793	-2.187
K5	2.231	-2.424
K7	3.215	-2.821
M0	3.436	-2.893
M1	4.214	-3.115
M2	5.185	-3.340
M3	8.379	-3.861
M4	12.301	-4.278
M5	18.426	-4.717
M6	37.497	-5.488

Note: The corresponding equation constant is equal to -(B-I).

With the star's I-band magnitude calculated, the next step is to determine which of the 1 million randomly pointed frames include the star in question. First, the star's right ascension and declination are converted into x, y, and z components of a unit vector. Each of

Table 2: I-Band : V-Band Flux Ratios and Corresponding Equation Constants

Spectral Type	I-Band:V-Band Flux Ratio	Corresponding Equation Constant
O5	0.364	0.358
09	0.364	0.356
B0	0.378	0.316
B3	0.435	0.165
B5	0.464	0.094
B8	0.480	0.057
A0	0.506	-0.000
A2	0.525	-0.040
A5	0.577	-0.144
A7	0.625	-0.230
F0	0.704	-0.358
F2	0.763	-0.447
F5	0.792	-0.486
F8	0.885	-0.607
G0	0.929	-0.661
G2	0.973	-0.711
G5	0.992	-0.731
G8	1.067	-0.811
K0	1.125	-0.867
K3	1.423	-1.123
K4	1.602	-1.252
K5	1.806	-1.382
K7	2.209	-1.601
M0	2.480	-1.726
M1	2.849	-1.877
M2	3.286	-2.032
M3	4.714	-2.424
M4	6.404	-2.756
M5	9.031	-3.129
M6	15.162	-3.692

Note: The corresponding equation constant is equal to -(V-I).

the random pointings are given as unit vectors. To reduce the total run time of the program, circular frames are assumed with areas equivalent to the square frames 1.24 degrees on a side that JMAPS will actually observe. With enough iterations, the effects of the different frame shapes diminish until they are negligible. The dot product of the random pointing unit vector and the star's unit vector is calculated, and if the result is less than the radius of the circular frame, assuming flat geometry, then the star is within the limits of the frame.

To further reduce the run time of the program, each star is only tested on reasonably nearby frames. A heap sort is implemented on the array containing the 1 million random pointings to sort each pointing by its x-component, so that each star will only be checked on pointing unit vectors with similar x-components.

3. Data

Tables 3, 4, and 5 show the total number of stars per I-band magnitude, binned from -0.5 to 0.5 etc., and the total cumulative number of stars of an I-band magnitude and all brighter magnitudes, meaning the number of stars brighter than I-band magnitude 0.5, etc from the V-band algorithm, the B-band algorithm, and SuperCOSMOS, respectively. These tables also show the average star counts per JMAPS frame, and Tables 4 and 5 also show the ratio of star counts to the corresponding results from the V-band algorithm.

Tables 6 and 7 show the distributions statistics, minimum, 2σ minimum, mode, mean, 2σ maximum, and maximum number of stars per simulated JMAPS frame as a function of non-cumulative I-band magnitude for the V-band algorithm and B-band Algorithm, respectively. Tables 8 and 9 show the same results as a function of cumulative I-band magnitude for the V-band algorithm and B-band Algorithm, respectively.

4. Analysis

We find good agreement in the results displayed in Tables 3, 4, and 5. In particular, the similarity between Tables 3 and 4, which show I-band magnitude distributions of stars from the V-band algorithm and B-band algorithm respectively, show that reddening did not alter the overall results of the study, which was anticipated to be a problem in TM 09-06.

Notable differences between the first two algorithms and SuperCOSMOS include lower star counts in SuperCOSMOS for I-band magnitudes brighter than 6 and dimmer than 14, except for the 17^{th} magnitude for which SuperCOSMOS has a higher star count. SuperCOSMOS shows roughly 14,000 stars of an I-band magnitude 6 and brighter, while the B and V-band algorithms both show around 28,000. A difference of 14,000 stars spread over the entire sky means a difference of about 0.5 stars per average JMAPS frame. At an I-band magnitude of 14, SuperCOSMOS reports around 34 million stars while the B and V-band algorithms both show about 55 million. An I-band magnitude of 14 corresponds to V-band magnitudes up to 17, which exceeds the limitations of 2MASS and NOMAD, increasing the amount of errors. More importance should be placed on the SuperCOSMOS data for I-band magnitudes less than 6 and greater than 14 because it comes from observational data as opposed to theoretical calculations from 2MASS and NOMAD data that may be compromised by inherent catalog inaccuracies and limitations.

Discrepancies at the brighter magnitudes may not be resolvable, but at the dim end they are. At the bright end, differences in the results are difficult to interpret because the method SuperCOSMOS used to derive those magnitudes is unclear. Photographic images for such stars would be unmeasurable, and no other source is described. Given the unknown systematic trends in magnitudes for bright stars in the SuperCosmos survey, greater reliance should be placed on the numbers from the B and V-band algorithms. All the I-band data from SuperCOSMOS was taken from the UK Science Research Council Near-Infrared Survey of the Galactic Plane (Hartley & Dawe 1981) or the National Geographic Society Palomar Observatory Sky Survey II (Reid et. al 1991), complete to I-band magnitudes of 19 and 19.5, respectively. The B and V-band algorithms rely on NOMAD data of stars with B and V-band magnitudes down to the 17^{th}, approaching the limits of accuracy in the catalog. As a result, greater reliance should be placed on the numbers from SuperCOSMOS for I-band magnitudes fainter than 14^{th}.

The agreement between all three results for I-band magnitudes between 6 and 14 indicates the accuracy of these results, and further corroborates the results from the JMAPS frame simulations. That is, because the overall magnitude tabulations between all three methods agree, the frame simulation results from the B and V-band algorithms can be assumed to be accurate even though those simulations could not be replicated with Super-

Table 3: V-band Algorithm Stars and Cumulative Stars per I-Band Magnitude

I-Band Magnitude	Stars	Stars Per Average JMAPS Frame	Cumulative Stars	Cumulative Stars per Average JMAPS Frame
0	19	0.0007	19	0.0007
1	27	0.0010	46	0.0017
2	86	0.0032	132	0.0049
3	493	0.01838	625	0.0233
4	1922	0.0716	2547	0.0949
5	6356	0.237	8903	0.3319
6	19515	0.727	28418	1.0593
7	55831	2.081	84249	3.1404
8	151037	5.630	235286	8.770
9	390613	14.56	625899	23.33
10	970092	36.16	1595991	59.49
11	2445778	91.17	4041769	150.7
12	6370379	237.5	10412148	388.1
13	14997172	559.0	25409320	947.1
14	30127027	1123.0	55536347	2070.1
15	52752620	1966.0	108288967	4036.5
16	47869103	1784.0	156158070	5820.8
17	1135421	42.32	157293491	5863.1

Table 4: B-band Algorithm Stars and Cumulative Stars per I-Band Magnitude

I-Band Magnitude	Stars	Stars Per Average JMAPS Frame	Star Count Ratio to the V-band Algorithm	Cumulative Stars	Cumulative Stars per Average JMAPS Frame	Cumulative Star Count Ratio to the V-band Algorithm
0	18	0.001	0.947	18	0.001	0.947
1	25	0.001	0.926	43	0.002	0.935
2	82	0.003	0.953	125	0.005	0.947
3	492	0.018	0.998	617	0.023	0.987
4	1912	0.071	0.995	2529	0.094	0.993
5	6345	0.237	0.998	8874	0.331	0.997
6	19487	0.726	0.999	28361	1.057	0.998
7	55747	2.078	0.998	84108	3.135	0.998
8	150791	5.621	0.998	234899	8.756	0.998
9	390009	14.538	0.998	624908	23.294	0.998
10	968563	36.103	0.998	1593471	59.397	0.998
11	2442797	91.056	0.999	4036268	150.452	0.999
12	6364794	237.249	0.999	10401062	387.701	0.999
13	14984533	558.551	0.999	25385595	946.252	0.999
14	30111971	1122.428	1.000	55497566	2068.680	0.999
15	52714686	1964.948	0.999	108212252	4033.628	0.999
16	47794012	1781.529	0.998	156006264	5815.157	0.999
17	1115809	41.592	0.983	157122073	5856.750	0.999

Table 5: SuperCOSMOS Stars and Cumulative Stars per I-Band Magnitude

I-Band Magnitude	Stars	Stars Per Average JMAPS Frame	Star Count Ratio to the V-band Algorithm	Cumulative Stars	Cumulative Stars per Average JMAPS Frame	Cumulative Star Count Ratio to the V-band Algorithm
0	5	0.000	0.263	5	0.000	0.263
1	124	0.005	4.593	129	0.005	2.804
2	149	0.006	1.733	278	0.010	2.106
3	170	0.006	0.345	448	0.017	0.717
4	478	0.018	0.249	926	0.035	0.364
5	2465	0.092	0.388	3391	0.126	0.381
6	11288	0.421	0.578	14679	0.547	0.517
7	49096	1.830	0.879	63775	2.377	0.757
8	176085	6.564	1.166	239860	8.941	1.019
9	550702	20.528	1.410	790562	29.468	1.263
10	1280075	47.715	1.320	2070637	77.183	1.297
11	2599729	96.905	1.063	4670366	174.089	1.156
12	4941883	184.209	0.776	9612249	358.298	0.923
13	9074860	338.267	0.605	18687109	696.565	0.735
14	16120290	600.886	0.535	34807399	1297.451	0.627
15	27714450	1033.060	0.525	62521849	2330.511	0.577
16	47047753	1753.712	0.983	109569602	4084.223	0.702
17	77196211	2877.500	67.989	186765813	6961.724	1.187

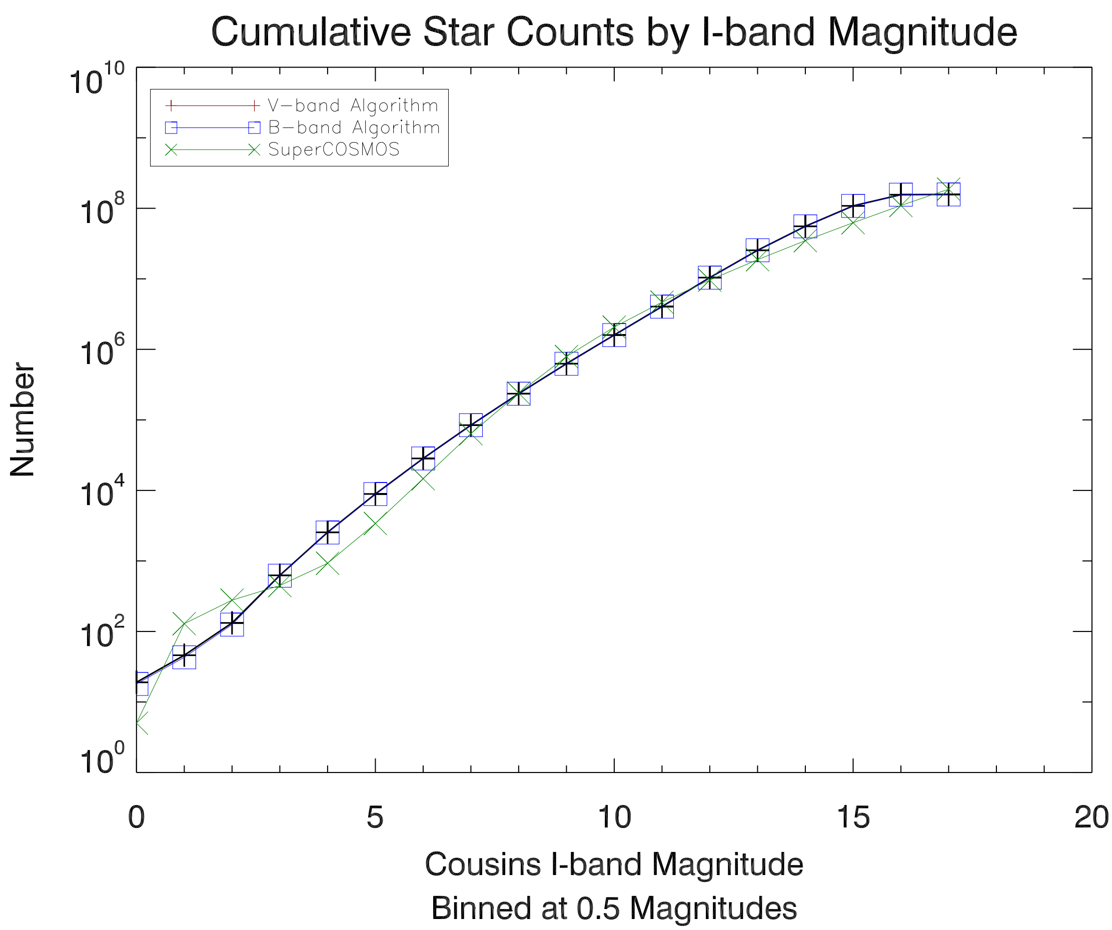

Figure 1: Cumulative Star Counts by I-band Magnitude from the V-band Algorithm, B-Band Algorithm, and SuperCOSMOS

Table 6: V-band Algorithm Stars per Simulated JMAPS Frame
as a function of I-Band Magnitude

I-Band Magnitude	Min. Stars per Frame	2 σ Min. Stars per Frame	Mode Stars per Frame	Average Stars per Frame	2 σ Max Stars per Frame	Max. Stars per Frame
0	0	0	0	0.002	0	1
1	0	0	0	0.001	0	1
2	0	0	0	0.003	0	1
3	0	0	0	0.018	0	2
4	0	0	0	0.071	1	5
5	0	0	0	0.237	2	7
6	0	0	0	0.725	3	11
7	0	0	1	2.080	7	21
8	0	0	3	5.622	16	44
9	0	2	6	14.539	46	118
10	0	6	12	36.117	142	516
11	3	14	22	91.044	447	3186
12	0	32	46	237.193	1316	12560
13	0	74	95	558.438	3276	28159
14	0	128	155	1122.498	6951	49242
15	0	195	262	1966.351	11942	48825
16	0	130	196	1781.341	10746	22757
17	0	0	2	41.628	312	1498

Note: This data shows statistics based on the first set of 1 million randomly pointed JMAPS frames.

Table 7: B-band Algorithm Stars per Simulated JMAPS Frame
as a function of I-Band Magnitude

I-Band Magnitude	Min. Stars per Frame	2 σ Min. Stars per Frame	Mode Stars per Frame	Average Stars per Frame	2 σ Max Stars per Frame	Max. Stars per Frame
0	0	0	0	0.010	0	5
1	0	0	0	0.001	0	1
2	0	0	0	0.019	0	2
3	0	0	0	0.077	1	4
4	0	0	0	0.233	2	8
5	0	0	0	0.748	3	11
6	0	0	1	2.042	7	21
7	0	0	2	5.334	18	51
8	0	1	5	13.920	55	180
9	0	4	9	34.846	171	840
10	1	11	18	88.396	515	4589
11	3	25	36	210.033	1360	13415
12	0	59	75	464.151	3100	27627
13	0	114	132	929.992	5970	51753
14	0	173	229	1568.821	9331	49608
15	0	157	230	1996.446	12131	30010

Note: This data shows statistics based on the first set of 1 million randomly pointed JMAPS frames.

Table 8: V-band Algorithm Cumulative Stars per Simulated JMAPS Frame as a function of I-Band Magnitude

I-Band Magnitude	Min. Stars per Frame	2 σ Min. Stars per Frame	Mode Stars per Frame	Average Stars per Frame	2 σ Max Stars per Frame	Max. Stars per Frame
0	0	0	0	0.002	0	1
1	0	0	0	0.003	0	1
2	0	0	0	0.006	0	2
3	0	0	0	0.024	1	2
4	0	0	0	0.095	1	6
5	0	0	0	0.332	2	8
6	0	0	0	1.057	4	16
7	0	0	2	3.137	9	35
8	0	1	5	8.759	23	71
9	0	5	11	23.298	68	181
10	3	14	21	59.415	208	646
11	12	31	44	150.459	652	3822
12	22	67	91	387.652	1965	16365
13	23	145	176	946.091	5255	43864
14	23	278	315	2068.589	12205	91541
15	28	479	614	4034.939	24178	122652
16	31	625	827	5816.280	33435	123482
17	31	628	830	5857.909	33626	123496

Note: This data shows statistics based on the first set of 1 million randomly pointed JMAPS frames.

Table 9: B-band Algorithm Cumulative Stars per Simulated JMAPS Frame as a function of I-Band Magnitude

I-Band Magnitude	Min. Stars per Frame	2 σ Min. Stars per Frame	Mode Stars per Frame	Average Stars per Frame	2 σ Max Stars per Frame	Max. Stars per Frame
0	0	0	0	0.010	0	5
1	0	0	0	0.011	0	5
2	0	0	0	0.029	1	5
3	0	0	0	0.106	1	6
4	0	0	0	0.340	2	9
5	0	0	0	1.087	4	15
6	0	0	2	3.129	10	32
7	0	1	4	8.463	26	73
8	0	4	9	22.384	79	221
9	2	10	17	57.230	249	1058
10	9	24	33	145.625	763	5624
11	23	52	69	355.658	2124	18450
12	25	114	138	819.809	5222	46077
13	27	233	279	1749.800	11203	95690
14	31	412	458	3318.621	20577	121332
15	32	583	772	5315.067	31485	123458
16	32	627	701	5832.078	33488	123514
17	32	628	830	5857.909	33626	123528

Note: This data shows statistics based on the first set of 1 million randomly pointed JMAPS frames.

COSMOS data. Important parameters that are unchanged include the magnitude JMAPS can rely on for guide stars at the 2σ level, which remains the 10^{th} magnitude.

5. Conclusions

Because of the good agreement between all three results, we conclude that reddening did not significantly alter the results of TM 09-06. We conclude that the stellar distributions from the B-band algorithm and the SuperCOSMOS data corroborate those from TM 09-06 for stars with I-band magnitudes 6-14. For stars with an I-band magnitude brighter than 6^{th} or dimmer than 14^{th}, SuperCOSMOS shows fewer stars than our simulations. Because the SuperCOSMOS method of determining magnitudes brighter than $6th$ is unclear, more emphasis should be placed on the results from the B and V-band algorithms for I-band magnitudes brighter than 6^{th}. Because the B and V-band algorithms use B and V-band magnitudes as dim as 17^{th} from NOMAD, which is beyond the catalog's limits of accuracy, to calculate I-band magnitudes dimmer than 14^{th}, more reliance should be put on the results from SuperCOSMOS for stars with an I-band magnitude dimmer than 14^{th}.

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